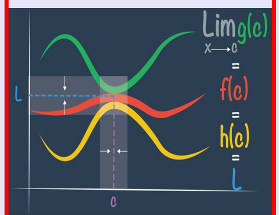
Calculus I

Lecture 15



Feb 19-8:47 AM

Evaluate

1) $$\lim_{x\to\infty} \frac{x^3 - x + 2}{(2x-1)(x^2+x+1)} = \lim_{x\to\infty} \frac{x^3 - x + 2}{2x^3 + x^2 + x - 1} = \frac{\infty}{\infty}$$

I.F.

Divide by x^3

$$= \lim_{x\to\infty} \frac{1 - \frac{1}{x^2} + \frac{2}{x^3}}{2 + \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}}$$

$\frac{1}{x^2} \to 0$, $\frac{2}{x^3} \to 0$, $\frac{1}{x} \to 0$, $\frac{1}{x^2} \to 0$, $\frac{1}{x^3} \to 0$

$$= \boxed{\frac{1}{2}}$$

Jan 29-10:13 AM

2) $\lim_{x\to\infty}\sqrt{\dfrac{4x^2-1}{25x^2-1}}$

$= \sqrt{\lim_{x\to\infty}\dfrac{4x^2-1}{25x^2-1}} = \sqrt{\lim_{x\to\infty}\dfrac{4-\cancel{\frac{1}{x^2}}\to 0}{25-\cancel{\frac{1}{x^2}}\to 0}}$

Divide by x^2

$= \sqrt{\dfrac{4}{25}} = \boxed{\dfrac{2}{5}}$

Jan 29-10:18 AM

3) $\lim_{x\to-\infty}\dfrac{\sqrt{4x^2+1}}{3x-1}$

Divide by x, $x\to-\infty$, $x=-\sqrt{x^2}$

$= \lim_{x\to-\infty}\dfrac{\frac{\sqrt{4x^2+1}}{x}}{\frac{3x-1}{x}} = \lim_{x\to-\infty}\dfrac{\frac{\sqrt{4x^2+1}}{-\sqrt{x^2}}}{\frac{3x}{x}-\frac{1}{x}}$

$= \lim_{x\to-\infty}\dfrac{-\sqrt{\frac{4x^2+1}{x^2}}}{3-\frac{1}{x}} = \lim_{x\to-\infty}\dfrac{-\sqrt{4+\cancel{\frac{1}{x^2}}\to 0}}{3-\cancel{\frac{1}{x}}\to 0}$

$= \dfrac{-\sqrt{4}}{3} = \boxed{\dfrac{-2}{3}}$

Jan 29-10:24 AM

4) $\lim_{x\to\infty}\left[\sqrt{4x^2+10x}-2x\right]=\infty-\infty$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{4x^2+10x}-2x\right)\left(\sqrt{4x^2+10x}+2x\right)}{\sqrt{4x^2+10x}+2x}$$

$$=\lim_{x\to\infty}\frac{\cancel{4x^2}+10x-\cancel{4x^2}}{\sqrt{4x^2+10x}+2x}=\lim_{x\to\infty}\frac{10x}{\sqrt{4x^2+10x}+2x}=\frac{\infty}{\infty}$$

Divide by x, as $x\to\infty$, $x=\sqrt{x^2}$

$$=\lim_{x\to\infty}\frac{\frac{10x}{x}}{\sqrt{\frac{4x^2}{x^2}+\frac{10x}{x^2}}+\frac{2x}{x}}$$

$$=\lim_{x\to\infty}\frac{10}{\sqrt{4+\cancel{\frac{10}{x}}\to 0}+2}=\frac{10}{\sqrt{4}+2}=\frac{10}{4}=\boxed{\frac{5}{2}}$$

Jan 29-10:30 AM

5) $\lim_{x\to\infty}\frac{\sin^4 x}{\sqrt{x}}$

$-1\leq \sin A\leq 1$

$0\leq \sin^4 x\leq 1$

Divide by $\sqrt{x}$

$$\frac{0}{\sqrt{x}}\leq\frac{\sin^4 x}{\sqrt{x}}\leq\frac{1}{\sqrt{x}}$$

$$0\leq\frac{\sin^4 x}{\sqrt{x}}\leq\frac{1}{\sqrt{x}}$$

$\lim_{x\to\infty} 0=0$

By S.T. $\lim_{x\to\infty}\frac{\sin^4 x}{\sqrt{x}}=0$

$\lim_{x\to\infty}\frac{1}{\sqrt{x}}=0$

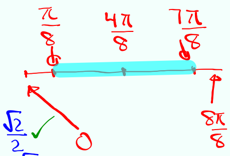
Jan 29-10:39 AM

$f(x) = \cos 2x$, $\left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$

check all conditions of the Rolle's thrm, then find all numbers c that satisfy the conclusion of Rolle's thrm.

1) $f(x)$ cont. on $\left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$

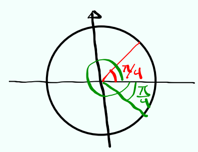
2) $f(x)$ diff. on $\left(\frac{\pi}{8}, \frac{7\pi}{8}\right)$



3) $f\left(\frac{\pi}{8}\right) = \cos 2\left(\frac{\pi}{8}\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ ✓

$f\left(\frac{7\pi}{8}\right) = \cos 2\left(\frac{7\pi}{8}\right) = \cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$ ✓

$\frac{7\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = 2\pi - \frac{\pi}{4}$



Conclusion

$f'(c) = 0$

$-\sin 2c = 0$

$\sin 2c = 0$

$\sin A = 0$

$0, \pi, 2\pi$

$2c = 0 \quad c = 0$

$2c = \pi \quad \boxed{c = \frac{\pi}{2}} = \frac{4\pi}{8}$

$2c = 2\pi \quad c = \pi = \frac{8\pi}{8}$

Jan 29-10:46 AM

$f(x) = \sqrt[3]{x}$, $[0, 1]$

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

verify all conditions of the MVT, then find all numbers c in $(0,1)$ that satisfy the conclusion of MVT.

1) $f(x)$ is Cont. $[0, 1]$

2) $f(x)$ is diff. $(0, 1)$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f'(c) = \frac{f(1) - f(0)}{1 - 0}$

$\frac{1}{3\sqrt[3]{c^2}} = \frac{1 - 0}{1 - 0}$

$\frac{1}{3\sqrt[3]{c^2}} = 1$

$3\sqrt[3]{c^2} = 1$

$\sqrt[3]{c^2} = \frac{1}{3}$

$c^2 = \left(\frac{1}{3}\right)^3$

$c^2 = \frac{1}{27}$

$c = \frac{1}{\sqrt{27}}$

$= \frac{1}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{c = \frac{\sqrt{3}}{9}}$

$\frac{\sqrt{3}}{9}$ is in $(0, 1)$

Jan 29-11:00 AM

Given $f'(1)=5$

Is $f(x)$ continuous at $x=1$? Explain

Since $f'(1)$ is defined

$f(x)$ is diff. at $x=1$.

If $f(x)$ is diff. at $x=a$, then

it is Cont. at $x=a$.

Yes, Since it is diff. at $x=1$.

Jan 29-11:10 AM

Prove if $f(x)$ is diff. at $x=a$, then $f(x)$ is cont. at $x=a$.

$$\lim_{x\to a} f(x) = \lim_{x\to a}\left[f(x)-f(a)+f(a)\right]$$

$$=\lim_{x\to a}\left[f(x)-f(a)\right] + \lim_{x\to a} f(a)$$

$$=\lim_{x\to a} \frac{f(x)-f(a)}{x-a}\cdot(x-a) + f(a)$$

$$=\boxed{\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}\cdot\boxed{\lim_{x\to a}(x-a)} + f(a)$$

$$= f'(a)\cdot(\underbrace{a-a}_{0}) + f(a)$$

$$= f(a)$$

we just showed $\lim_{x\to a} f(x) = f(a)$

$\therefore$ $f(x)$ is cont. at $x=a$.

Jan 29-11:15 AM

$f(x) = 12 + 4x - x^2$, $[0,5]$

1) find $f(0)$ & $f(5)$

$f(0) = 12$, $f(5) = 7$

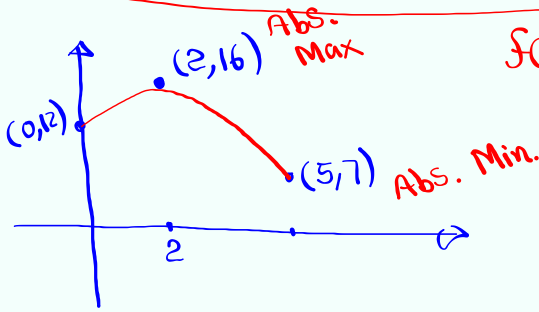
2) Evaluate $f(x)$ at x-values in the $[0,5]$ where $f'(x) = 0$.

$f'(x) = 4 - 2x$

$f'(x) = 0$ $x = 2$

$f(2) = 12 + 4(2) - 2^2$

$= \boxed{16}$



Jan 29-11:21 AM

$f(x) = 5 + 54x - 2x^3$, $[0,4]$

1) Evaluate $f(x)$ at the endpoints of the interval

$f(0) = 5$, $f(4) = 93$

2) Evaluate $f(x)$ at x-values in that interval where $f'(x) = 0$

$f'(x) = 54 - 6x^2$

$f'(x) = 0$

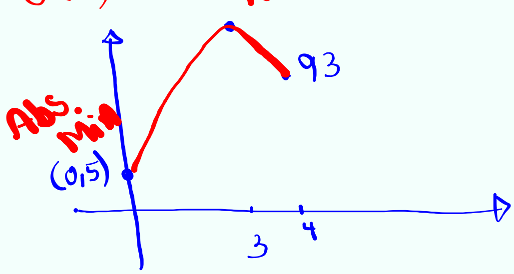
$54 - 6x^2 = 0$

$6x^2 = 54$

$x^2 = 9$

$x = \pm 3$

$f(3) = 113$ Abs. Max



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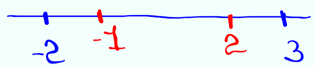
Find Abs. Max & Abs. Min. of

$f(x) = 2x^3 - 3x^2 - 12x + 1$ over $[-2, 3]$

$f(-2) = -3$

$f(3) = -8$



$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

Divide by 6

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x - 2 = 0 \qquad x + 1 = 0$

$x = 2 \qquad x = -1.$

$f(-1) = 8$

$f(2) = -19$

Abs. Max at $(-1, 8)$

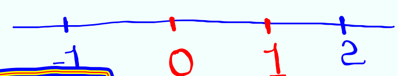
Abs. Min. at $(2, -19)$

Jan 29-11:38 AM

Find Abs. Max & Abs. Min of $f(x) = (x^2 - 1)^3$

Cont. ✓

on $[-1, 2]$.



$\boxed{f(-1) = 0}$, $\boxed{f(2) = 27}$

$f'(x) = 3(x^2 - 1)^2 \cdot 2x = 6x(x^2 - 1)^2$

$f'(x) = 0 \qquad x = 0 \qquad x^2 - 1 = 0$

$x = 1 \qquad x = -1$

$\boxed{f(0) = -1}$ $\boxed{f(1) = 0}$

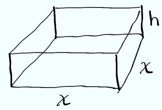
Abs. Max at $(2, 27)$

Abs. Min at $(0, -1)$

Jan 29-11:48 AM

A box has a square base, open top with volume $32{,}000\ cm^3$.

Find dimensions of the box that minimizes the materials to make it.



$V = LWH$

$= x\,x\,h$

$\boxed{32000 = x^2 h} \rightarrow h = \dfrac{32000}{x^2}$

$h = \dfrac{32000}{40^2} = \dfrac{\cancel{32000}^{\,20}}{\cancel{1600}} = 20$

Materials: base + 4 sides

$x \cdot x + 4\,xh$

$M(x) = x^2 + 4x \cdot \dfrac{32000}{x^2}$

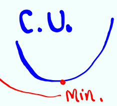
$M(x) = x^2 + \dfrac{128000}{x}, \quad x > 0$

Find x to minimize $M(x)$

$M(x) = x^2 + 128000x^{-1}$

$M'(x) = 2x - 128000x^{-2}$

$M''(x) = 2 + 256000x^{-3} = 2 + \dfrac{256000}{x^3} > 0$



$M'(x) = 0$

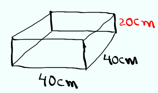
$2x - \dfrac{128000}{x^2} = 0$

$2x^3 - 128000 = 0$

$x^3 - 64000 = 0$

$x^3 = 64000$

$x = \sqrt[3]{64000} = 40$



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